

Rayleigh Scattering

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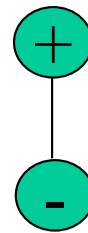
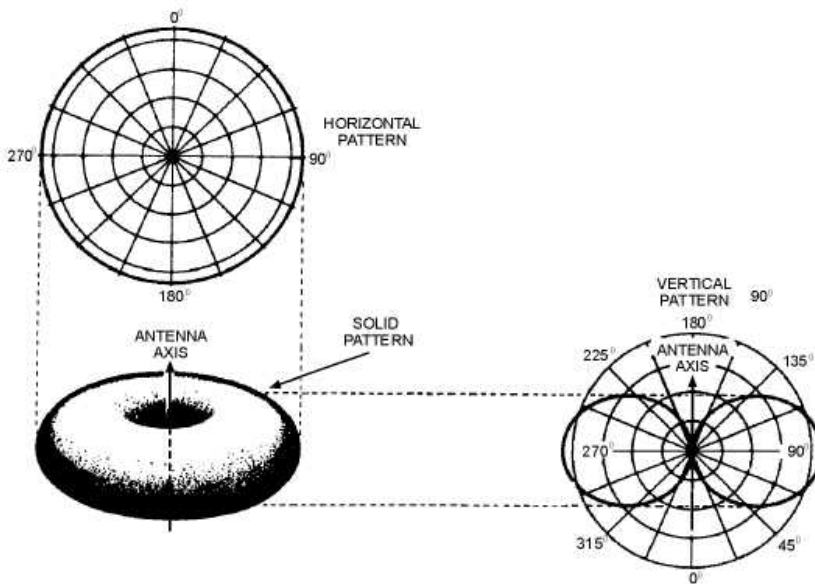


Rayleigh Scattering

- Rayleigh scatter is elastic scattering from atoms and molecules.
- Elastic means there is no final change in quantum state and thus the frequency of the scattered light is the same as the incoming light.
- It is a non-resonant process meaning the incoming photon energy $h\nu$ is not equal to a difference between two energy levels.
- It occurs because the incoming light induces a dipole moment in the atom or molecule which then re-radiates.
- Mostly used for density or temperature measurements.

Electric Field Induced Radiation

- Classical dipole



$$\vec{F} = -e\vec{E}$$

$$E \propto \cos(\omega t)$$

$$\text{Dipole Moment } \vec{\mu} = \sum_i q_i \vec{r}_i$$

$$\vec{E}_{emitted} \propto \vec{\mu}^2$$

Dipole moment can be induced:

$$\vec{\mu} = \alpha \vec{E}$$

α is the Polarizability

Rayleigh Scattering

- It is applicable when the radius of the scattering particle is much less than the wavelength of incoming light.
- It is proportional to one over the wavelength to the fourth power, i.e.

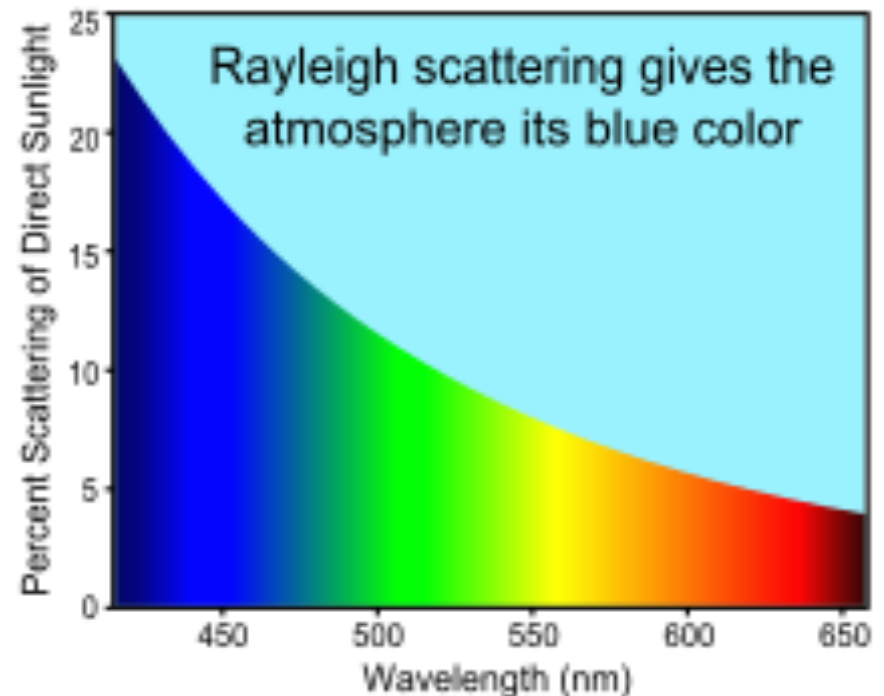
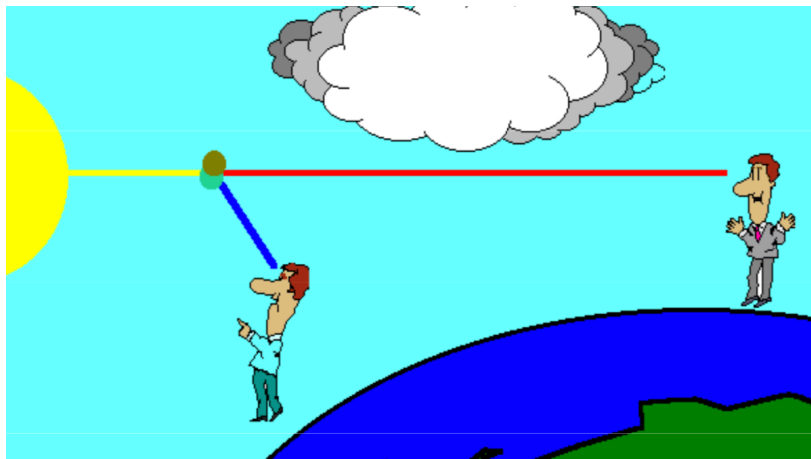
$$I = I_0 \frac{16\pi^4 r^6 \left(\frac{n^2 - 1}{n^2 + 2} \right)^2}{\lambda^4 L} \cos^2(\theta)$$

- where n is the index of refraction.
- One can define a “cross section” for Rayleigh scattering:

$$\sigma_s = \frac{2\pi^5}{3} \frac{(2r)^6}{\lambda^4} \left(\frac{n^2 - 1}{n^2 + 2} \right)^2$$

Why is the Sky Blue and Sunsets Red?

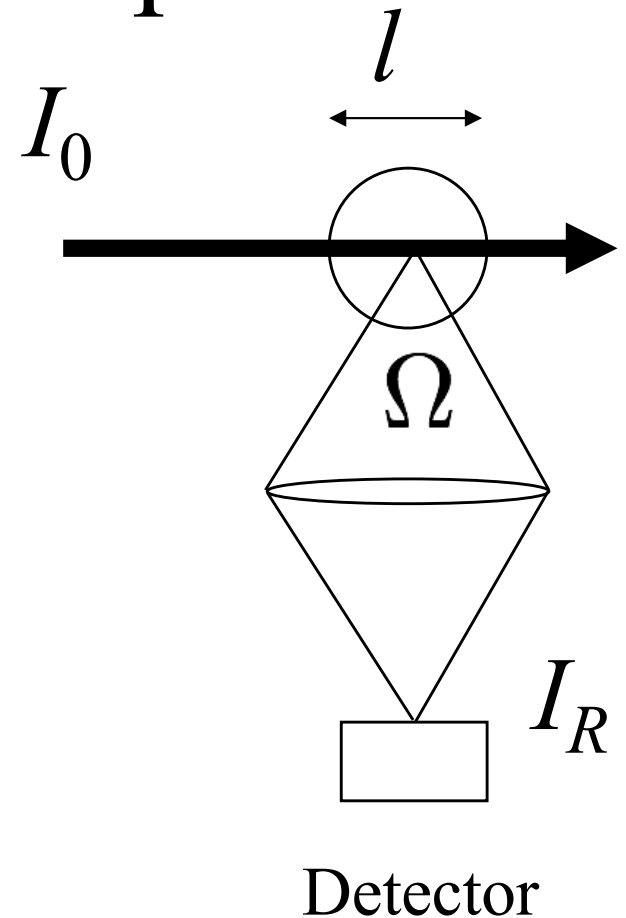
- Because of the wavelength dependency, higher frequencies, corresponding to shorter wavelengths, are preferentially scattered.



A Measurement Setup

$$I_R = I_0 N l \Omega \epsilon \left(\frac{d\sigma}{d\Omega} \right)_{eff}$$

- I_0 Incident radiant power
- I_R Collected radiant power
- Ω Solid angle
- l Pathlength
- ϵ Optical Efficiency
- N Number concentration



$$\left(\frac{d\sigma}{d\Omega} \right)_{eff}$$

Rayleigh cross section for gas mixture

Rayleigh Cross Sections

Species	Cross Section (cm ² /sr)
Ar	4.45x10 ⁻²⁷
N ₂	5.10x10 ⁻²⁷
CO	6.19x10 ⁻²⁷
CH ₄	12.47x10 ⁻²⁷
N ₂ O	15.90x10 ⁻²⁷
CO ₂	12.4x10 ⁻²⁷

Direct measurement of the Rayleigh scattering cross section in various gases,
Maarten Snee, Wim Ubachs, Journal of Quantitative Spectroscopy & Radiative
Transfer 92 (2005) 293–310